

### Remarks

In this final office action, the Examiner rejects claims 2-9 under 35 U.S.C. §112 as failing to comply with the written description requirement. The Examiner believes that the specification indicates an element “Visco-Elastic Connector” is not the same as the term, “Elastic Polymer Connector block” as stated in claims 8 and 9. The Examiner also believes that the specification does not support the new term and raises new matter.

Accordingly, the claims have been amended to reintroduce the visco-elastic term. It is respectfully submitted that this term simply defines a rubber-like material which has both elastic and damping qualities. The definition of visco-elasticity is illustrated in the attached encyclopedia description. The difference between a pure elastic metal spring and a visco-elastic rubber-like polymer is best seen in the stress-strain diagram on page 2, with the visco-elastic material exhibiting the hysteresis or damping effects. It is believed that the amendment which reintroduces the visco-elastic term to the claims introduced no new matter. The specification specifically indicates on page 3, line 31, that the visco-elastic material may be a polymer. Therefore, the term polymer introduces no new matter in the claims. The claims in their current form distinguish the cited references. The visco-elastic polymer connector is attached in series to the inboard end of the lower arm and the connector elastically bends, thereby allowing the arm to pivot relative to the bicycle as the bicycle leans into a turn.

A visco-elastic material is one that exhibits both viscous and elastic characteristics. Such a material can act both as a spring and as a damper, allowing it to cushion applied force and absorb energy. Typical materials that possess these characteristics are natural and synthetic rubbers (elastomers or polymers).

The visco-elastic connector described:

- a. Is a simple solid compound, therefore no cross-section illustrating its internal structure is required;

b. Is connected to the lower arm, and the upper bracket, by (screw) fasteners (Item 4), which screw into mating sockets molded into the visco-elastic material. These screws can be seen in Figures 4 and 3.

Alternatively, the upper bracket the lower arm and the visco-elastic connector could be moulded together in one operation, without the need for fasteners.

c. Has a shape that allows more up and down movement than fore and aft movement as shown on Figure 1 - some fore and aft movement is desirable to absorb bumps but it needs to be limited for stability;

d. Is the pivot, the spring and the damper all in the one item as shown on Figure 1;

The visco-elastic material to be used would be chosen after careful consideration of properties such as elasticity, resilience, durability, appearance and cost to best suit the application. More than one formulation or size connector may be required to cover the age and/or weight range of riders.

It is believed that the description provides sufficient information to satisfy the requirement to enable someone skilled in the art to make the device.

It is respectfully requested that the Examiner telephone the undersigned if there are any remaining outstanding issues so that any concerns may be resolved by an Examiner's amendment in the case and promptly passed to issue.

Please charge any fees or credit any overpayments as a result of the filing of this paper to our Deposit Account No. 02-3978.

Respectfully submitted,

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Enclosure

# Viscoelasticity

From Wikipedia, the free encyclopedia  
(Redirected from Visco-elastic).

**Viscoelasticity**, also known as **anelasticity**, describes materials that exhibit both viscous and elastic characteristics. Viscous materials, like honey, resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain instantaneously when stretched and just as quickly return to their original state once the stress is removed. Viscoelastic materials have elements of both of these properties and, as such, exhibit time dependent strain. Whereas elasticity is usually the result of bond stretching along crystallographic planes in an ordered solid, viscoelasticity is the result of the diffusion of atoms or molecules inside of an amorphous material [1].

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## Background

In the nineteenth century, physicists such as Maxwell, Boltzmann, and Kelvin researched and experimented with creep and recovery of glasses, metals, and rubbers [2]. Viscoelasticity was further examined in the late twentieth century when synthetic polymers were engineered and used in a variety of applications [2]. Viscoelasticity calculations depend heavily on the viscosity variable,  $\eta$ . The inverse of  $\eta$  is also known as fluidity,  $\phi$ . The value of either can be derived as a function of temperature or as a given value (ie for a dashpot) [1].

Depending on the change of strain rate versus stress inside a material the viscosity can be

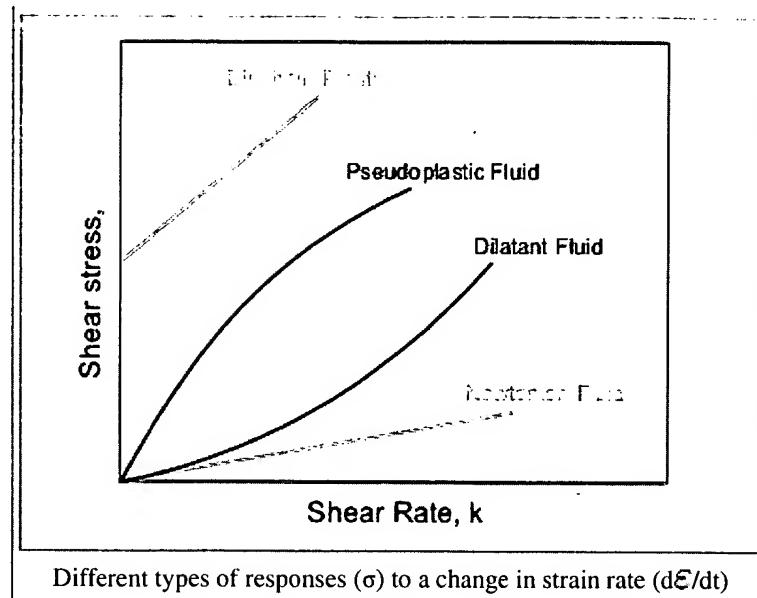
categorized as having a linear, non-linear, or plastic response. When a material exhibits a linear response it is categorized as a Newtonian material [1]. In this case the stress is linearly proportional to the strain rate. If the material exhibits a non-linear response to the strain rate, it is categorized as Non-Newtonian fluid. There is also an interesting case where the viscosity decreases as the strain rate increases. A material which exhibits this type of behavior is known as thixotropic [1]. In addition, when the stress is independent of this strain rate, the material exhibits plastic deformation [1]. Many viscoelastic materials exhibit rubber like behavior explained by the thermodynamic theory of polymer elasticity.

Some examples of viscoelastic materials include amorphous polymers, semicrystalline polymers, biopolymers, and metals at very high temperatures.

A viscoelastic material has the following properties:

- hysteresis is seen in the stress-strain curve.
- stress relaxation occurs: step constant strain causes decreasing stress
- creep occurs: step constant stress causes increasing strain

## Elastic Behavior Versus Viscoelastic Behavior

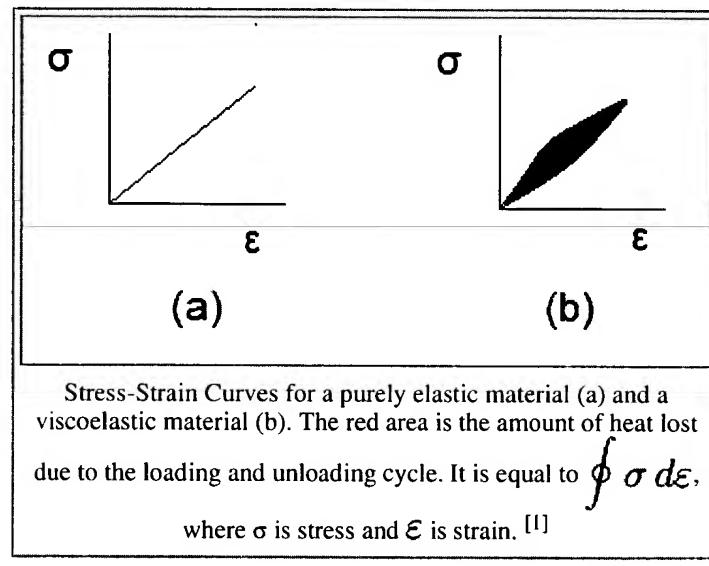


Unlike purely elastic substances, a viscoelastic substance has an elastic component and a viscous component. The viscosity of a viscoelastic substance gives the substance a strain rate dependent on time<sup>[1]</sup>. Purely elastic materials do not dissipate energy (heat) when a load is applied, then removed<sup>[1]</sup>. However, a viscoelastic substance loses energy when a load is applied, then removed. Hysteresis is observed in the stress-strain curve, with the area of the loop being equal to the energy lost during the loading cycle<sup>[1]</sup>. Since viscosity is the resistance to thermally activated plastic deformation, a viscous material will lose energy through a loading cycle. Plastic deformation results in lost energy, which is uncharacteristic of a purely elastic material's reaction to a loading cycle<sup>[1]</sup>.

Specifically, viscoelasticity is a molecular rearrangement. When a stress is applied to a viscoelastic material such as a polymer, parts of the long polymer chain change position. This movement or rearrangement is called Creep. Polymers remain a solid material even when these parts of their chains are rearranging in order to accompany the stress, and as this occurs, it creates a back stress in the material. When the back stress is the same magnitude as the applied stress, the material no longer creeps. When the original stress is taken away, the accumulated back stresses will cause the polymer to return to its original form. The material creeps, which gives the prefix visco-, and the material fully recovers, which gives the suffix -elasticity<sup>[2]</sup>.

## Types of viscoelasticity

**Linear viscoelasticity** is when the function is separable in both creep response and load. All linear viscoelastic models can be represented by a Volterra equation connecting stress and strain:



$$\epsilon(t) = \frac{\sigma(t)}{E_{inst,creep}} + \int_0^t K(t-t')\sigma(t')dt'$$

or

$$\sigma(t) = E_{inst,relax}\epsilon(t) + \int_0^t F(t-t')\epsilon(t')dt'$$

where

- $t$  is time
- $\sigma(t)$  is stress
- $\epsilon(t)$  is strain
- $E_{inst,creep}$  and  $E_{inst,relax}$  are instantaneous elastic moduli for creep and relaxation
- $K(t)$  is the creep function
- $F(t)$  is the relaxation function

Linear viscoelasticity is usually applicable only for small deformations.

**Nonlinear viscoelasticity** is when the function is not separable. It is usually happens when the deformations are large or if the material changes its properties under deformations.

## Dynamic modulus

Viscoelasticity is studied using dynamic mechanical analysis. When we apply a small oscillatory strain and measure the resulting stress.

- Purely elastic materials have stress and strain in phase, so that the response of one caused by the other is immediate.
- In purely viscous materials, strain lags stress by a 90 degree phase lag.
- Viscoelastic materials exhibit behavior somewhere in the middle of these two types of material, exhibiting some lag in strain.

Complex Dynamic modulus  $G$  can be used to represent the relations between the oscillating stress and strain:

$$G = G' + iG''$$

where  $i = \sqrt{-1}$ ;  $G'$  is the *storage modulus* and  $G''$  is the *loss modulus*:

$$G' = \frac{\sigma_0}{\epsilon_0} \cos \delta$$

$$G'' = \frac{\sigma_0}{\epsilon_0} \sin \delta$$

where  $\sigma_0$  and  $\epsilon_0$  are the amplitudes of stress and strain and  $\delta$  is the phase shift between them.

## Constitutive Models of Linear Viscoelasticity

Viscoelastic materials, such as amorphous polymers, semicrystalline polymers, and biopolymers, can be modeled in order to determine their stress or strain interactions as well as their temporal dependencies. These models, which include the Maxwell model, the Kelvin-Voigt model, and the Standard Linear Solid Model, are used to predict a material's response under different loading conditions. Viscoelastic behavior is comprised of elastic and viscous components modeled as linear combinations of springs and dashpots, respectively. Each model differs in the arrangement of these elements, and all of these viscoelastic models can be equivalently modeled as electrical circuits. The elastic modulus of a spring is analogous to a circuit's resistance and the viscosity of a dashpot to a capacitor. The elastic components, as previously mentioned, can be modeled as springs of elastic constant  $E$ , given the formula:

$$\sigma = E\varepsilon$$

where  $\sigma$  is the stress,  $E$  is the elastic modulus of the material, and  $\varepsilon$  is the strain that occurs under the given stress, similar to Hooke's Law.

The viscous components can be modeled as dashpots such that the stress-strain rate relationship can be given as,

$$\sigma = \eta \frac{d\varepsilon}{dt}$$

where  $\sigma$  is the stress,  $\eta$  is the viscosity of the material, and  $d\varepsilon/dt$  is the time derivative of strain.

The relationship between stress and strain can be simplified for specific stress rates. For high stress states/short time periods, the time derivative components of the stress-strain relationship dominate. A dashpot resists changes in length, and in a high stress state it can be approximated as a rigid rod. Since a rigid rod cannot be stretched past its original length, no strain is added to the system<sup>[3]</sup>

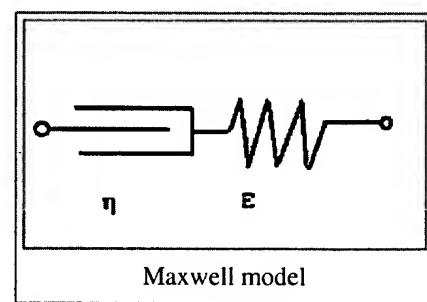
Conversely, for low stress states/longer time periods, the time derivative components are negligible and the dashpot can be effectively removed from the system - an "open" circuit. As a result, only the spring connected in parallel to the dashpot will contribute to the total strain in the system<sup>[3]</sup>

## Maxwell model

The Maxwell model can be represented by a purely viscous damper and a purely elastic spring connected in series, as shown in the diagram. The model can be represented by the following equation:

$$\frac{d\varepsilon_{Total}}{dt} = \frac{d\varepsilon_D}{dt} + \frac{d\varepsilon_S}{dt} = \frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt}.$$

The model represents a liquid (able to have irreversible deformations) with some additional reversible (elastic) deformations. If put under a constant strain, the stresses gradually relax. When a material is put under a constant stress, the strain has two components as per the Maxwell Model. First, an elastic component occurs instantaneously, corresponding to the spring, and relaxes immediately upon release of the stress. The second is a viscous component that grows with time as long as the stress is applied. The Maxwell model predicts that stress decays exponentially with time, which is accurate for most polymers. It is important to note limitations of such a model, as it is unable to predict creep in materials based on a simple dashpot and spring connected in series. The Maxwell model for creep or constant-stress



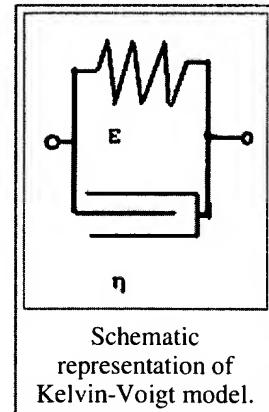
conditions postulates that strain will increase linearly with time. However, polymers for the most part show the strain rate to be decreasing with time<sup>[2]</sup>.

## Kelvin-Voigt model

The Kelvin-Voigt model, also known as the Voigt model, consists of a Newtonian damper and Hookean elastic spring connected in parallel, as shown in the picture. It is used to explain the stress relaxation behaviors of polymers. The constitutive relation is expressed as a linear first-order differential equation:

$$\sigma(t) = E\epsilon(t) + c \frac{d\epsilon(t)}{dt}$$

This model represents a solid undergoing reversible, viscoelastic strain. Upon application of a constant stress, the material deforms at a decreasing rate, asymptotically approaching the steady-state strain. When the stress is released, the material gradually relaxes to its undeformed state. At constant stress (creep), the Model is quite realistic as it predicts strain to tend to  $\sigma/E$  as time continues to infinity. Similar to the Maxwell model, the Kelvin-Voigt Model also has limitations. The model is extremely good with modelling creep in materials, but with regards to relaxation the model is much less accurate.



Schematic representation of Kelvin-Voigt model.

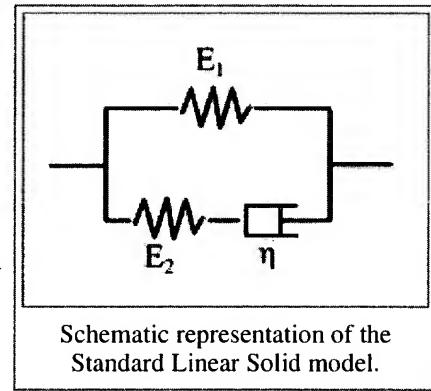
## Standard Linear Solid Model

The Standard Linear Solid Model effectively combines the Maxwell Model and a Hookean spring in parallel. A viscous material is modeled as a spring and a dashpot in series with each other, both of which are in parallel with a lone spring. For this model, the governing constitutive relation is:

$$\frac{d\epsilon}{dt} = \frac{\frac{E_2}{\eta} \left( \frac{\eta}{E_2} \frac{d\sigma}{dt} + \sigma - E_1 \epsilon \right)}{E_1 + E_2}$$

Under a constant stress, the modeled material will instantaneously deform to some strain, which is the elastic portion of the strain, and after that it will continue to deform and asymptotically approach a steady-state strain.

This last portion is the viscous part of the strain. Although the Standard Linear Solid Model is more accurate than the Maxwell and Kelvin-Voigt models in predicting material responses, mathematically it returns inaccurate results for strain under specific loading conditions and is rather difficult to calculate.



Schematic representation of the Standard Linear Solid model.

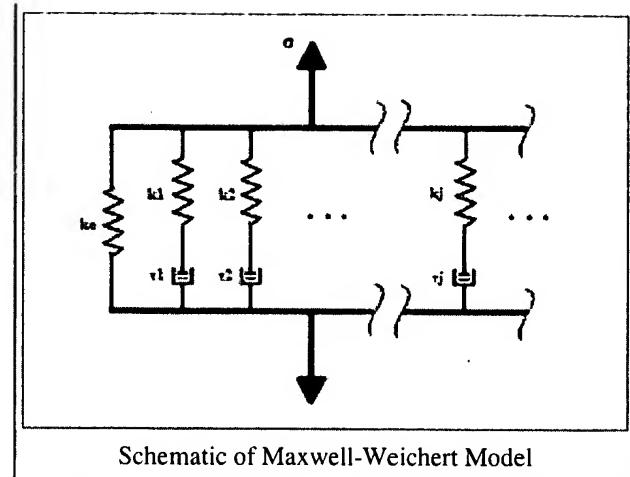
## Generalized Maxwell Model

The Generalized Maxwell also known as the Maxwell-Weichert model (after James Clerk Maxwell and Dieter

Weichert) is the most general form of the models described above. It takes into account that relaxation does not occur at a single time, but at a distribution of times. Due to molecular segments of different lengths with shorter ones contributing less than longer ones, there is a varying time distribution. The Weichert model shows this by having as many spring-dashpot Maxwell elements as are necessary to accurately represent the distribution. The Figure on the right represents a possible Weichert model<sup>[4]</sup>.

## Effect of Temperature on Viscoelastic Behavior

The secondary bonds of a polymer constantly break and reform due to thermal motion. Application of a stress favors some conformations over others, so the molecules of the polymer will gradually "flow" into the favored conformations over time. Because thermal motion is one factor contributing to the deformation of polymers, viscoelastic properties change with increasing or decreasing temperature. In most cases, the creep modulus, defined as the ratio of applied stress to the time-dependent strain, decreases with increasing temperature. Generally speaking, an increase in temperature correlates to a logarithmic decrease in the time required to impart equal strain under a constant stress. In other words, it takes less energy to stretch a viscoelastic material an equal distance at a higher temperature than it does at a lower temperature.



Schematic of Maxwell-Weichert Model

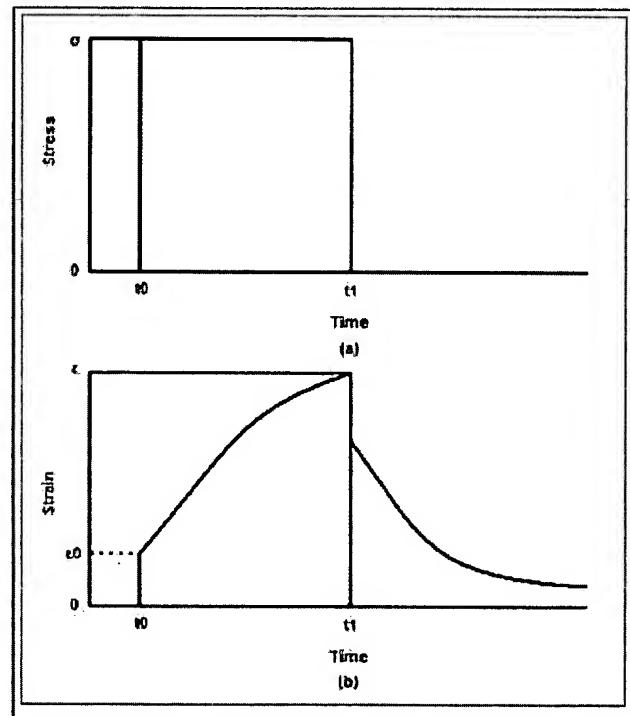
## Viscoelastic Creep

When subjected to a step constant stress, viscoelastic materials experience a time-dependent increase in strain. This phenomenon is known as viscoelastic creep.

At a time  $t_0$ , a viscoelastic material is loaded with a constant stress that is maintained for a sufficiently long time period. The material responds to the stress with a strain that increases until the material ultimately fails. When the stress is maintained for a shorter time period, the material undergoes an initial strain until a time  $t_1$ , after which the strain immediately decreases (discontinuity) then gradually decreases at times  $t > t_1$  to a residual strain.

Viscoelastic creep data can be presented by plotting the creep modulus (constant applied stress divided by total strain at a particular time) as a function of time [5]. Below its critical stress, the viscoelastic creep modulus is independent of stress applied. A family of curves describing strain versus time response to various applied stress may be represented by a single viscoelastic creep modulus versus time curve if the applied stresses are below the material's critical stress value.

Viscoelastic creep is important when considering long-term structural design. Given loading and temperature conditions, designers can choose materials that best suit component lifetimes.



Applied stress (a) and induced strain (b) as functions of time for a viscoelastic material.

## Measuring Viscoelasticity

Though there are many instruments that test the mechanical and viscoelastic response of materials, broadband viscoelastic spectroscopy (BVS) and resonant ultrasound spectroscopy (RUS) are more commonly used to test viscoelastic behavior because they can be used above and below ambient temperatures and are more specific to testing viscoelasticity. These two instruments employ a damping mechanism at various frequencies and time ranges with no appeal to time-temperature superposition [6]. Using BVS and RUS to study the mechanical properties of materials is important to understanding how a material exhibiting viscoelasticity will perform [6].

## References

1. <sup>a b c d e f g h i j</sup> Meyers and Chawla (1999): "Mechanical Behavior of Materials," 98-103.
2. <sup>a b c d</sup> McCrum, Buckley, and Bucknell (2003): "Principles of Polymer Engineering," 117-176.
3. <sup>a b</sup> Van Vliet, Krystyn J. (2006); "3.032 Mechanical Behavior of Materials", [1]
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## See also

- Polymer
- Creep
- Stress relaxation
- Hysteresis
- Biomaterial
- Biomechanics
- Rubber Elasticity
- Fractional calculus

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